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BPS D-branes on non-supersymmetric cycles

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Abstract: In certain regions of the moduli space of K3 and Calabi-Yau manifolds, D-branes wrapped on non-supersymmetric cycles may give rise to stable configurations. We show that in the orbifold limit, some of these stable configurations can be described by solvable boundary conformal field theories. The world-volume theory of N coincident branes of this type is described by a non-supersymmetric U(N) gauge theory. At the boundary of the region of stability, there are marginal deformations connecting the non-supersymmetric brane to a pair of D-branes wrapped on supersymmetric cycles. We also discuss various relationships between BPS and non-BPS D-branes of type II string theories.

Keywords: D-branes, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction and summary

Type IIA (IIB) string theory contains $2p$-dimensional ($(2p+1)$-dimensional) extended objects known as Dirichlet branes (D-branes), which are invariant under half of the space-time supersymmetry transformations of the theory, are charged under the gauge fields arising in the Ramond-Ramond (RR) sector of the theory [1], and are stable. Upon compactification of the type II string theory on a K3 surface or a Calabi-Yau three fold, we can get stable, supersymmetric branes in the compactified theory by wrapping these D-branes on various supersymmetric cycles [2] of the compact manifold. These branes can also be shown to be stable, and invariant under part of the supersymmetry transformations of the theory.

Generically K3 surfaces and Calabi-Yau 3-folds also contain topologically non-trivial cycles which are not supersymmetric, and we can consider configurations where a D-brane is wrapped around one such cycle. The stability of this configuration is not guaranteed by supersymmetry. Typically such a non-supersymmetric cycle is homologically equivalent to a sum of several supersymmetric cycles. Thus it is in principle possible for a D-brane wrapped on a non-supersymmetric cycle to decay into several D-branes, each wrapped on a supersymmetric cycle. Whether such a decay is energetically favourable depends on the values of various moduli parametrising the vacuum. By working in the orbifold limit of K3 and certain Calabi-Yau space we show that D-branes wrapped on non-supersymmetric cycles can be stable in certain regions of the moduli space. Furthermore, it is possible to describe such a wrapped

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brane by a solvable boundary conformal field theory. Beyond the region of stability, the state can decay into a pair of BPS states obtained by wrapping a D-brane on supersymmetric cycles. This is signalled by the appearance of a tachyonic mode on the brane world-volume. At the boundary of the region of stability, the tachyon represents an exactly marginal deformation of the boundary conformal field theory, and interpolates between the non-BPS state and a pair of BPS states.

The starting point in our analysis are the non-BPS D-branes of type II string theories discussed in [3, 4, 5, 6, 7]. In particular type IIA (IIB) string theory contains non-BPS D-$(2p+1)$ (D-$2p$) branes. These branes are unstable, as indicated by the presence of a tachyonic mode on the brane world-volume. We show that if we start with type IIA string theory on T^4, take the non-BPS D-string of type IIA string theory wrapped on a circle of the torus, and mod out the theory by a Z_2 transformation which changes the sign of all the coordinates of the torus, then the resulting configuration can be interpreted as a D-membrane of type IIA string theory, wrapped on a non-supersymmetric cycle of the K3 orbifold. In certain range of values of the radii of the compact directions, the spectrum of open strings on the brane is free from tachyons, and hence the brane is stable. Outside this range a tachyonic mode develops indicating the existence of a lower energy configuration with the same quantum numbers. This result can be generalized to branes of higher dimensions, and also to the case of Calabi-Yau manifold obtained by taking the $Z_2 \times Z_2$ orbifold of a six dimensional torus.

The paper is organised as follows. Throughout the paper we work in the limit of weak string coupling and restrict our analysis to open string tree level. In section 2 we analyse some aspects of type IIA D-string wrapped on a circle. This is a non-BPS configuration and has tachyonic modes. If the radius of the circle is R, and if x denotes the coordinate along the circle, then $T(x)$ has an expansion:

$$T(x) = \sum_{n=-\infty}^{\infty} T_n e^{inx/R} . \tag{1.1}$$

The nth mode has mass squared

$$m_n^2 = \frac{n^2}{R^2} - \frac{1}{2} , \tag{1.2}$$

in $\alpha' = 1$ unit. Thus at the critical radius $R = \sqrt{2}$, $T_{\pm 1}$ becomes massless. We show that the combination $(T_1 - T_{-1})$ represents an exactly marginal deformation at the critical radius and study the effect of switching on the vacuum expectation value of $(T_1 - T_{-1})$. This can be done using bosonization techniques similar to the ones discussed in [5]. The final result is that by switching on an appropriate vev of the tachyon field, the boundary conformal field theory describing the system can be reduced to the one describing a $D0 - \bar{D}0$ pair of type IIA string theory, situated at diametrically opposite points on the circle.

From eq. (1.1) we see that switching on $(T_1 - T_{-1})$ corresponds to a tachyon field configuration proportional to $\sin(x/R)$. This has been shown in fig. 1. This diagram suggests that switching on the tachyon vev of this form corresponds to the creation of a kink-antikink pair on the D-string. Combining this with the results stated in the last paragraph, we see that a kink (antikink) solution on a non-BPS D-string of type IIA string theory can be identified to a D0-brane (anti-D0-brane) of type IIA string theory.

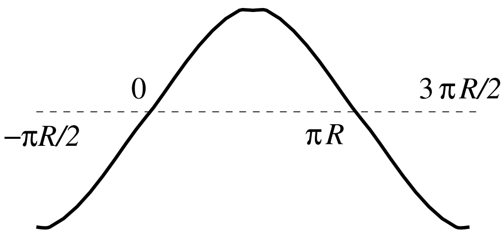

Figure 1: The tachyon background corresponding to $(T_1 - T_{-1})$ excitation.

If we make an $R \to (1/R)$ duality transformation on the compact coordinate, then type IIA string theory goes to type IIB string theory, the initial non-BPS D-string becomes the non-BPS D-particle, and the final D0-brane anti-D0-brane pair separated by πR becomes a D-string anti-D-string pair of type IIB with half a unit of Wilson line on one of them. This is precisely the system studied in [5], except that the final state here was the initial state there, and the initial state here was the final state there. Thus the results of section 2 could have been derived by running the analysis of [5] backwards. There is however a specific reason why we have chosen to carry out the analysis explicitly. Although [5] shows how, by starting with a D-string - anti-D-string configuration of IIB, we can produce a non-BPS D-particle via a series of marginal deformations, it was never shown explicitly that this D-particle is identical to the one described in [6]. Thus it is instructive to carry out the analysis backwards, *i.e.* start from a non-BPS D-particle of IIB as defined in [6], and find the series of marginal deformations that takes it to a D-string anti-D-string pair. This is precisely what is established by the analysis of section 2.

In section 3 we take the non-BPS D-string of type IIA compactified on a circle, and mod out the theory by a Z_2 transformation $\mathcal{I}_4$ which reverses the direction of the circle (which we denote by x^9) and also of three other directions x^6, x^7, x^8. This gives a non-BPS state in the orbifold theory. We show that the zero momentum mode of the tachyon is projected out, and so from eq. (1.2) we see that the spectrum is free from tachyons in the range $R \leq \sqrt{2}$. Thus this gives a stable non-BPS state in the theory.[1] In order to find a physical interpretation of this state, we note that $T_1 - T_{-1}$ survives the projection by $\mathcal{I}_4$, and hence at the critical radius $R = \sqrt{2}$ there still exists a marginal deformation which takes the present state to a D0-brane $-$ anti-D0-brane pair situated at the fixed points (planes) of the Z_2 transformation $\mathcal{I}_4$. These in turn can be interpreted as type IIA membranes wrapped on the 2-cycles associated with the blow up modes of the fixed points [8]. Since the stable non-BPS state below the critical radius has the same quantum numbers as the sum of the

[1]This configuration is related by T-duality to the stable non-BPS state discussed in refs. [3, 4].

quantum numbers of this pair of wrapped membranes, it is natural to interprete the non-BPS state as a membrane wrapped on a non-supersymmetric 2-cycle which is homologically equivalent to the sum of the two 2-cycles associated with the two blow up modes. As the radius changes from below $\sqrt{2}$ to above $\sqrt{2}$, it becomes energetically favourable for the brane wrapped on the non-supersymmetric cycle to break up into a pair of branes wrapped on supersymmetric cycles.

If we take the directions $x^6, \ldots x^8$ also to be compact, then the theory under study is type IIA string theory on a K3 orbifold, and the non-BPS state describes a membrane wrapped on a non-supersymmetric cycle of K3. This construction can be easily generalized to a D-$(2p+2)$-brane (D-$(2p+1)$-brane) of type IIA (IIB) string theory wrapped on a non-supersymmetric 2-cycle of K3 by starting with an initial configuration where we have a non-BPS D-$(2p+1)$ (D-$2p$) brane of IIA (IIB) with one direction along x^9 and other directions along the non-compact directions.

In section 4 we study the world-volume theory of N coincident branes of this type. This requires computing the spectrum of massless open string states with ends on the D-branes, and can be done by standard techniques. Thus, for example, if we consider the D5-brane of type IIB wrapped on a non-supersymmetric 2-cycle of K3, we get a non-supersymmetric U(N) gauge theory in (3+1) dimensions with four massless Majorana fermions and two massless scalars in the adjoint representation of the gauge group,

In section 5 we consider some generalisations of the results of section 3. In particular we consider the case where we have the same K3 orbifold, but the starting configuration is a non-BPS D3-brane with all three directions tangential to the torus. Using $R \to (1/R)$ duality transformations we argue that after modding out the theory by $\mathcal{I}_4$ this configuration represents a membrane wrapped on a new 2-cycle of K3. We also construct non-BPS states on a Calabi-Yau manifold obtained by compactifying two more directions (x^4 and x^5), and modding out the theory further by a new Z_2 transformation $\mathcal{I}_4'$ which reverses the direction of x^4, x^5, x^6 and x^7. We include appropriate shifts in the definition of $\mathcal{I}_4'$ so that neither $\mathcal{I}_4'$ nor $\mathcal{I}_4\mathcal{I}_4'$ has any fixed point. (This model was discussed in [9].) By starting with an appropriate $\mathcal{I}_4$ and $\mathcal{I}_4'$ invariant combination of non-BPS D-branes wrapped on various directions of T^6, we construct branes wrapped on non-supersymmetric 2- and 3-cycles of the Calabi-Yau manifold.

Section 6 is somewhat outside the main theme of this paper. In this section we discuss the interrelation between various supersymmetric and non-supersymmetric D-branes of type II string theories. These relations take the form of two step descent relations where we start from a brane-antibrane pair and end up with a single BPS D-brane. The first set of relations is obtained as follows. Let us consider a D-$2p$ $-$ anti-D-$2p$ brane pair of type IIA string theory. There is a tachyonic excitation on this system [10] such that the tachyonic ground state corresponds to vacuum configuration [11, 12, 13]. If instead we take a tachyonic kink solution on this brane

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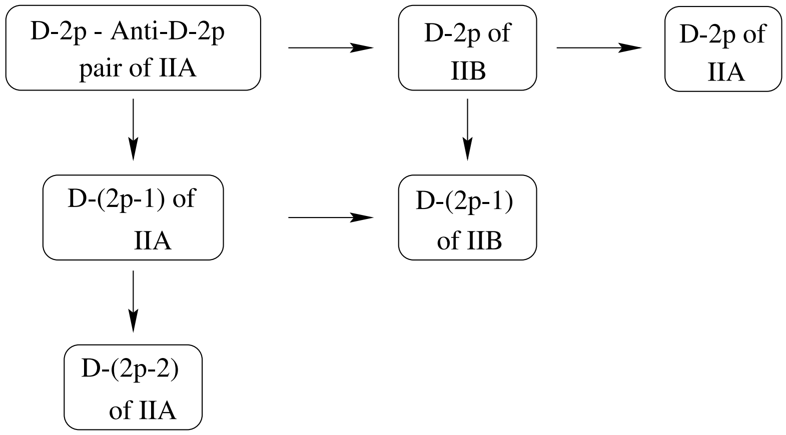

Figure 2: The relationship between different D-branes. The horizontal arrows represent the result of modding out the theory by $(-1)^{F_L}$, where the vertical arrows represent the effect of constructing a tachyonic kink solution.

antibrane pair, then the analysis of [5] shows that it describes a non-BPS D-$(2p-1)$ brane of IIA. This system also has a tachyonic excitation. If we consider a kink solution on the D-$(2p-1)$ brane associated with this tachyon, then according to the analysis of section 2, the result is a BPS D-$(2p-2)$-brane of IIA. Similar result holds if we start with a D-$(2p+1)$-brane $-$ anti-D-$(2p+1)$ brane pair of type IIB string theory. In this case the end product is a D-$(2p-1)$ brane of IIB.

Another set of relations which we derive in this section is as follows. Let us again start from a D-$2p$-brane $-$ anti-D-$2p$-brane pair of IIA. But this time, instead of considering the tachyonic kink solution on this pair, we mod out the theory by $(-1)^{F_L}$, where $(-1)^{F_L}$ acts as -1 on all the Ramond sector states on the left-moving part of the world-sheet of the fundamental string, and leaves the other sectors unchanged. We show that the result is a non-BPS D-$2p$-brane of IIB. Upon further modding out the theory by $(-1)^{F_L}$ we get a BPS D-$2p$-brane of IIA.

Thus we have two sets of descent relations relating the various BPS and non-BPS D-branes. These relations have been summarized in fig. 2.

2. Type IIA D-string on a circle

In this section we shall start with a non-BPS D-string of type IIA string theory wrapped on a circle, and identify a series of marginal deformations which map it to a D0-brane anti- D0-brane pair, situated at diametrically opposite points on the circle. If we make a T-duality transformation along the circle, then the starting configuration is a D-particle of type IIB string theory, and the final configuration is a D-string anti-D-string pair, with half a unit of Wilson line along one of the strings. It was already shown in [5] that these two configurations are related by marginal deformation; so one might wonder why we cannot simply take the result of that paper. To this end, note that in the analysis of [5], we started from the D-string anti-D-string pair of IIB, and identified a series of marginal deformations whose end product was a D-particle on a circle. Although there is a great deal of evidence that this D-particle is the same as the D-particle defined in refs. [4, 6], this was never proved conclusively. The analysis of this section starts from the T-dual of the D-particle described in [4, 6] and identifies that marginal deformation that takes it to the T-dual of the D-string anti-D-string pair of IIB. Thus the result of this section can be taken to be further evidence for the equivalence between the D-particle of [4, 6] and of [5].

The non-supersymmetric D-string of type IIA string theory can be described in a manner similar to the one used in describing the non-supersymmetric D-particle of type IIB string theory [6]. If we take the D-string to lie along the 9th direction, then we put Dirichlet boundary condition on the coordinates $X^1, \ldots X^8$, and Neumann boundary condition on the coordinates X^0, X^9. However, unlike an ordinary D-string of type IIB string theory, the spectrum of open strings with both ends on the type IIA D-string contains Fock space states which are $(-1)^F$ even as well as $(-1)^F$ odd; with the $(-1)^F$ even states carrying a Chan Paton (CP) factor equal to the 2×2 identity matrix I, and the $(-1)^F$ odd states carrying Chan Paton factor σ_2, where σ_i $(1 \le i \le 3)$ are the three Pauli matrices.[2] We use the standard convention that $(-1)^F$ acts as -1 on the Neveu-Schwarz (NS) sector ground state of the open string. This system clearly has a tachyonic excitation coming from the NS sector ground state in the Chan Paton sector σ_2.

We shall be analysing the case where the 9th direction has been compactified on a circle of radius R_9 so that the momentum k_9 along this direction is quantized in units of $1/R_9$. For simplicity of notation we shall denote X^9, R_9 and k_9 by X, R and k respectively. In the $\alpha' = 1$ unit, the tachyon has mass^2 = $(-1/2)$. Thus the nth mode T_n of the tachyon, defined through the expansion:

$$T(x) = \sum_{n=-\infty}^{\infty} T_n e^{i\frac{n}{R}x}\,, \tag{2.1}$$

has effective mass^2:

$$m_n^2 = \frac{n^2}{R^2} - \frac{1}{2}\,. \tag{2.2}$$

From this we see that at the critical radius

$$R = \sqrt{2} \tag{2.3}$$

$T_{\pm 1}$ becomes massless. The vertex operators for $T_{\pm 1}$ are given as follows. Let ψ, $\widetilde{\psi}$ denote the right- and the left-moving components of the world-sheet fermion associated

[2] A slightly different description, which is extremely useful for discussing spinor states, has been given in [7]. But we shall not use this description here.

with the 9th direction, X_L, X_R denote the left- and the right-moving components of the scalar field $X(= X_L + X_R)$, and $\widetilde{\Phi}, \Phi$ denote the left- and the right-moving components of the bosonized ghost [15]. Then, for NS sector states, the fields X, Φ, $\widetilde{\Phi}$, ψ, $\widetilde{\psi}$ satisfy the boundary conditions:[3]

$$(X_L)_B = (X_R)_B \equiv X_B/2, \qquad \psi_B = \widetilde{\psi}_B\,, \qquad \Phi_B = \widetilde{\Phi}_B\,, \tag{2.4}$$

where the subscript B denotes the values of various fields at the boundary of the world-sheet. In the -1 picture [15], the vertex operator of $T_{\pm 1}$ at the critical radius is given by:

$$V_{\pm}^{(-1)} = -e^{-\Phi_B} e^{\pm\frac{i}{\sqrt{2}}X_B} \otimes \sigma_2\,. \tag{2.5}$$

The overall $-$ sign in the above equation is a matter of convention. In the zero picture these vertex operators take the form:

$$V_{\pm}^{(0)} = \mp i\psi_B e^{\pm\frac{i}{\sqrt{2}}X_B} \otimes \sigma_2\,. \tag{2.6}$$

As in [5], we shall now fermionize the world-sheet scalar X in order to express the vertex operators in a simpler form. The bose-fermi relation takes the form:

$$e^{i\sqrt{2}X_R} = \frac{1}{\sqrt{2}}(\xi + i\eta)\,, \qquad e^{i\sqrt{2}X_L} = \frac{1}{\sqrt{2}}(\widetilde{\xi} + i\widetilde{\eta})\,. \tag{2.7}$$

We can find another representation of the same conformal field theory by rebosonizing the fermions as follows:

$$\frac{1}{\sqrt{2}}(\xi + i\psi) = e^{i\sqrt{2}\phi_R}\,, \qquad \frac{1}{\sqrt{2}}(\widetilde{\xi} + i\widetilde{\psi}) = e^{i\sqrt{2}\phi_L}\,. \tag{2.8}$$

ϕ represents a free bosonic field with radius $\sqrt{2}$. There is a third representation in which we use a slightly different rebosonization:

$$\frac{1}{\sqrt{2}}(\eta + i\psi) = e^{i\sqrt{2}\phi'_R}\,, \qquad \frac{1}{\sqrt{2}}(\widetilde{\eta} + i\widetilde{\psi}) = e^{i\sqrt{2}\phi'_L}\,, \tag{2.9}$$

where ϕ' is another scalar field of radius $\sqrt{2}$. We can also relate the fermionic and the bosonic U(1) currents as follows:

$$\psi\xi = i\sqrt{2}\partial\phi_R\,, \qquad \eta\xi = i\sqrt{2}\partial X_R\,, \qquad \psi\eta = i\sqrt{2}\partial\phi'_R\,. \tag{2.10}$$

There are also similar relations involving the left-moving currents.

Since X satisfies Neumann boundary condition $(X_L)_B = (X_R)_B$ at the boundary of the world sheet, this translates to the Neumann boundary condition on the fermions:

$$\xi_B = \widetilde{\xi}_B\,, \qquad \eta_B = \widetilde{\eta}_B\,. \tag{2.11}$$

[3]These boundary conditions are written in the coordinate system where the open string world sheet is represented as the upper half plane.

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From (2.4), (2.8), (2.9) and (2.11) we see that ϕ and ϕ' both satisfy Neumann boundary condition at both ends:

$$(\phi_R)_B = (\phi_L)_B \equiv \frac{1}{2}\phi_B\,, \qquad (\phi'_R)_B = (\phi'_L)_B \equiv \frac{1}{2}\phi'_B\,. \tag{2.12}$$

However, as argued in [5], in the Ramond sector ϕ, ϕ' satisfy Neumann boundary condition at one end and Dirichlet boundary condition at the other end.

Using the bosonization relations (2.7)- (2.10) we can now express the tachyon vertex operators in a simpler form. However, before we do that, we need to address a subtle issue related to bosonization. Analysing (2.7) carefully we see that the left-hand side of this equation commutes with an operator carrying odd world-sheet fermion number (*e.g.* ψ), whereas the right hand side, being a fermionic operator, anti-commutes with an operator of odd world-sheet fermion number. In order to resolve this problem, we need to assign 'cocycle' factors [14] as follows. To every operator which is odd under $(-1)^F$ we assign a cocycle factor of τ_3, and to every operator which carries odd unit of momentum along x^9, we assign a cocycle factor of τ_1. Here τ_i $(1 \le i \le 3)$ are Pauli matrices. Thus eq. (2.7) now takes the form:

$$e^{i\sqrt{2}X_R} = \frac{1}{\sqrt{2}}(\xi + i\eta)\otimes\tau_1\,, \qquad e^{i\sqrt{2}X_L} = \frac{1}{\sqrt{2}}(\tilde{\xi} + i\tilde{\eta})\otimes\tau_1\,. \tag{2.13}$$

This gives an extra $-$ sign in the commutation relation of the right hand side of this equation with the $(-1)^F$ odd states. Thus both sides of the equation now has the same commutation relations.[4]

Combining this rule with the rule for the Chan Paton factor, we see that if n denotes the number of units of x^9 momentum carried by a vertex operator, then we need to assign Chan Paton and cocycle factors depending on the values of $(-1)^n$ and $(-1)^F$ given in table 1. Note that the 4×4 matrices I and Σ_i defined in this table satisfy the same algebra as the 2×2 identity matrix and the Pauli matrices.

$(-1)^F$	$(-1)^n$	CP$\otimes$cocycle factor
even	even	$I\otimes I \equiv I$
even	odd	$I\otimes\tau_1 \equiv \Sigma_3$
odd	even	$\sigma_2\otimes\tau_3 \equiv \Sigma_2$
odd	odd	$\sigma_2\otimes\tau_2 \equiv \Sigma_1$

Table 1: CP$\otimes$cocycle factors.

Thanks to these bosonization rules, and eqs. (2.5), (2.6), we can express

$$V_T \equiv \frac{1}{\sqrt{2}}(V_+ - V_-)\,, \tag{2.14}$$

representing the vertex operator correponding to the tachyonic mode $(T_1 - T_{-1})$, as:

$$V_T^{(-1)} = e^{-\Phi_B}\eta_B\otimes\Sigma_1, \qquad V_T^{(0)} = \psi_B\xi_B\otimes\Sigma_1\,. \tag{2.15}$$

[4]One might wonder why similar cocycle factors were not necessary in the analysis of [5]. There all allowed vertex operators on the D-string anti- D-string system were even under $(-1)^F h$, where $h=-1$ $(+1)$ for vertex operators carrying odd (even) units of x^9 momentum. The cocycle factors for a $(-1)^F$ even, h even state is I, whereas that for a $(-1)^F$ odd, h odd state is τ_2. Since these commute with each other, we could ignore them in the analysis of [5].

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We shall concentrate our attention on the NS sector states, for which ϕ satisfies Neumann boundary condition. In this case, using eq. (2.10) and the definition of ϕ_B given in eq. (2.12) we can rewrite $V_T^{(0)}$ as:

$$V_T^{(0)} = \frac{i}{\sqrt{2}}\partial\phi_B \otimes \Sigma_1 \,. \tag{2.16}$$

From here on, the analysis proceeds exactly as in [5]. Switching on vev of the tachyon amounts to switching on a Wilson line along the ϕ direction. If as in [5] we label the tachyon vev by a parameter α with appropriate normalization, then at $\alpha = 1$ the spectrum takes the following form. In the sectors with Chan Paton $\otimes$ cocycle factor I or Σ_1, the spectrum remains unchanged from its form at $\alpha = 0$, whereas in the sector with Chan Paton $\otimes$ cocycle factor Σ_2 and Σ_3, the allowed Fock space states have opposite $(-1)^F(-1)^n$ quantum numbers compared to the spectrum at $\alpha = 0$. Since from table 1 we see that in these sectors, $(-1)^F(-1)^n = -1$ in the absence of tachyon vev, we conclude that at $\alpha = 1$ these states have $(-1)^F(-1)^n = 1$. On the other hand, in the sectors corresponding to I and Σ_1, $(-1)^F(-1)^n = 1$ in the absence of tachyon vev, and hence they remain so even at $\alpha = 1$. Thus we can conclude that at $\alpha = 1$, all states have $(-1)^F(-1)^n = 1$; and furthermore, every state in the Fock space with $(-1)^F(-1)^n = 1$ appears twice in the spectrum. This in turn implies that there is no tachyonic state in the spectrum at this value of α, as the possible tachyon state, coming from the NS sector ground state, has $(-1)^F = -1$ and $n = 0$, and hence has $(-1)^F(-1)^n = -1$.

Following [5] we can now study the effect of increasing the radius R beyond its critical value $\sqrt{2}$. As in [5] one finds that for $R > \sqrt{2}$ the $\alpha = 1$ point represents a local minimum of the tachyon potential, and the effect of increasing the radius of the x^9 direction to $R = L\sqrt{2}$ at $\alpha = 1$ can be related to decreasing the radius of the ϕ' coordinate to $\sqrt{2}/L = 2/R$ in the absence of tachyon vev. Denoting by ϕ'_D the coordinate dual to ϕ', one finds that the ϕ'_D coordinate has radius $R/2$; and there is Dirichlet boundary condition along the ϕ'_D direction. Since the ϕ'_D radius goes to ∞ as $R \to \infty$, it is natural to interprete ϕ'_D as the 9th direction. Its associated world-sheet fermion is $\xi, \widetilde{\xi}$. Denoting by F_{new} the new world-sheet fermion number under which $\xi, \widetilde{\xi}$ are odd, and $\psi, \widetilde{\psi}, \eta, \widetilde{\eta}$ are even, and by n'_ϕ the number of units of ϕ' momentum or equivalently ϕ'_D winding, we have the relation:

$$(-1)^F(-1)^n = (-1)^{F_{new}}(-1)^{n'_\phi} \,. \tag{2.17}$$

This can be easily verified by studying the action of both sides on various fields. Thus we see that the spectrum of open string states contains Fock space states with $(-1)^{F_{new}}(-1)^{n'_\phi} = 1$, with each state in the Fock space satisfying this relation appearing twice in the spectrum. Note that since ϕ'_D has radius $R/2$, n'_ϕ unit of winding charge corresponds to a total winding charge of $\pi R n'_\phi$.

Let us now compare this with the spectrum of open strings in type IIA string theory compactified on a circle of radius R, with a D0-brane sitting at $x^9 = a$, and an anti-D0-brane sitting at $x^9 = \pi R + a$ where a is some constant. In this case the Fock space will contain two copies of $(-1)^F$ even states from open string with both ends on the D0-brane or both ends on the anti-D0-brane. Also these states will carry a total winding charge which is integral multiple of $2\pi R$, *i.e.* even multiple of πR. On the other hand open strings with one end on the D0-brane and the other end on the anti-D0-brane correspond to Fock space states with $(-1)^F = -1$, carry winding charge which is odd multiple of πR, and also come in pairs due to two different orientations of the string. If we denote by $\pi R n'_\phi$ the total winding charge, we see that the full spectrum consists of two copies of the Fock space states for which $(-1)^F(-1)^{n'_\phi} = 1$. This is exactly identical to the spectrum obtained in the previous paragraph. Thus we conclude that the marginal deformation of the wrapped D-string of the type IIA string theory that we have discussed in this section takes us to a D-particle anti-D-particle pair of type IIA string theory situated at diametrically opposite points of the circle.

This result can be interpreted in the following way. Switching on the real component of the tachyon field corresponding to $(T_1 - T_{-1})$ corresponds to a background tachyon field of the form:

$$T(x) \propto \sin\frac{x}{R}\,. \tag{2.18}$$

As suggested by fig. 1, this corrsponds to the creation of a kink-antikink pair separated by πR. On the other hand our analysis shows that switching on $T_1 - T_{-1}$ deformation takes us to a D0-$\bar{\rm D}$0 brane pair, separated by πR. This suggests that we should identify the kink (anti-kink) solution on the type IIA D-string as the type IIA D-particle (anti-D-particle).

This result has the following consequence. By T-dualizing the analysis of ref. [5] one can easily see that the type IIA D-string can be regarded as a tachyonic kink on the membrane-antimembrane pair. The excitations on this kink has a new tachyonic mode, due to the fact that the tachyon field on the membrane anti-membrane pair is complex, and hence the kink solution is unstable. Now we see that the kink solution associated with this new tachyonic mode is stable, and represents the D-particle of type IIA string theory. On the other hand a simple topological analysis shows that the double kink, representing the tachyonic kink on the kink solution on the membrane anti-membrane pair, is nothing but the tachyonic vortex on the membrane-antimembrane pair discussed in [5, 7]. This establishes the claim that the tachyonic vortex on the membrane antimembrane pair of type IIA string theory represents a D-particle. This can be easiliy generalized to show the equivalence of the vortex solution on a D$(p+2)$-brane anti- D$(p+2)$-brane to a Dp brane – a result which has been suggested in ref. [3] and used recently to show that D-brane charges take values on the K-theory of space-time [7].

3. Type IIA D-string on an orbifold

In this section we shall analyse a Z_2 orbifold of the system considered in section 2. The Z_2 group will be generated by an element $\mathcal{I}_4$ which reverses the sign of the 9th direction as well as three other directions which we shall take to be along x^6, x^7 and x^8. Also for definiteness, we shall take the fixed planes of this transformation to be located at $x^6 = x^7 = x^8 = 0$, $x^9 = 0, \pi R$. In order to see the fate of the non-supersymmetric D-string along the 9th direction under this orbifold operation, we need to know how $\mathcal{I}_4$ acts on the open string states with ends lying on the D-string. This, in turn, requires knowing the action of $\mathcal{I}_4$ on the oscillators, the momenta, as well as on the vacuum. The action on the oscillators and the momentum components is straightforward; those associated with $X^6, \ldots X^9$ and their world-sheet fermionic partners change sign; whereas those associated with $X^0, \ldots X^5$ and their fermionic partners do not change sign. The action on the vacuum needs to be determined separately in each Chan Paton sector. First of all in the Chan Paton sector I, the mode corresponding to translation of the D-string along the X^i direction for $1 \le i \le 5$ must be even under $\mathcal{I}_4$ since it has a non-vanishing 2-point function with the $0i$ component of the space-time metric which is even under $\mathcal{I}_4$. Since the $X^1, \ldots X^5$ oscillators do not change sign under $\mathcal{I}_4$, this shows that $\mathcal{I}_4$ must leave the vacuum in this sector invariant. In order to determine the action of $\mathcal{I}_4$ on the vacuum in the Chan Paton sector σ_2, let us note that if A_μ denotes the RR sector vector field of the type IIA string theory, then the two point function of A_9, and the tachyonic open string state in the Chan Paton sector σ_2, carrying zero momentum along $x^6, \ldots x^9$, is non-vanishing. (This can be seen by a computation very similar to one done in ref. [7] for non-supersymmetric D0-brane.) Since A_9 is odd under $\mathcal{I}_4$, this shows that the tachyonic ground state with zero $x^6, \ldots x^9$ momentum is odd under $\mathcal{I}_4$. In other words, the vacuum in the Chan Paton sector σ_2 is odd under $\mathcal{I}_4$.

From this we see that the tachyon carrying zero momentum along x^9 is projected out when we mod out the theory by $\mathcal{I}_4$. For non-zero x^9 momentum n/R, the modes which survive are:

$$T_n - T_{-n} \,. \tag{3.1}$$

Since the $n = 0$ mode is absent, from eq. (2.2) we now see that below the critical radius $R = \sqrt{2}$, there are no tachyonic modes. Thus the configuration is stable in this region. On the other hand above the critical radius, this configuration becomes unstable due to the appearance of a tachyonic mode $T_1 - T_{-1}$. As we have seen in section 2, this signals the possibility of the decay of the wrapped D-string into a pair of $D0$-branes at diametrically opposite points on the x^9 axis. In order to see what this final configuration corresponds to in the orbifold theory, we need to have more precise information about the locations of the D0-brane - anti-D0-brane pair. Since modding out by $\mathcal{I}_4$ breaks translation invariance along x^9, it is nolonger sufficient to just say that they are separated by a distance πR. To this end, note that since we have a single D0-brane and a single anti- D0-brane, the only way to get an $\mathcal{I}_4$ invariant configuration is to have one of them located at $x^9 = 0$, and the other one located at $x^9 = \pi R$. This can also be seen explicitly from fig. 1. The location of the 0-branes after tachyon condensation can be taken to be the places where the tachyon field vanishes, since these are the places where the fundamental property of the D-brane – that open strings can end there – remains intact. From fig. 1 we see that when the combination $T_1 - T_{-1}$ is switched on, these points are located at $x^9 = 0$ and $x^9 = \pi R$. Thus these are the locations of the D0-brane and the anti-D0-brane after tachyon condensation.

The effect of modding out such a configuration by $\mathcal{I}_4$ was studied in [8]. The result was that these correspond to membranes wrapped on the collapsed 2-cycles associated with the orbifold singularities. Naively one would think that these configurations would have vanishing mass; however this does not happen since on each of these collapsed 2-cycles there is half a unit of flux of the rank two anti-symmetric tensor field arising in the NSNS sector of the closed string [16]. Thus we see that when $R > \sqrt{2}$, our original configuration becomes unstable against decay into a pair of membranes, wrapped around the pair of 2-cycles associated with the orbifold fixed planes at $(x^5, \ldots x^9) = (0, 0, 0, 0)$ and $(0, 0, 0, \pi R)$. This clearly suggests that the stable configuration for $R < \sqrt{2}$ that we had started with has the interpretation of a membrane that wraps around a 2-cycle which is homologically identical to the sum of the 2-cycles associated with the two fixed points. This is not a supersymmetric cycle [2], as the state obtained by wrapping a membrane on it is not a BPS state. Nevertheless a membrane wrapped on it is stable in some region of the moduli space.

This interpretation can also be seen by constructing the boundary state [17, 18] corresponding to the original configuration. Before the orbifold projection the boundary state is given by:

$$|\theta, U\rangle_{NSNS}\,, \tag{3.2}$$

where $|\theta, U\rangle_{NSNS}$ has been defined in eq.(3.4) of [3]. Here θ is the Wilson line associated with the U(1) gauge field living on the D-string. Note that although the boundary states for D-string were defined in [3] for type IIB D-strings, we can use the NSNS sector components of the same boundary state to describe the D-string of type IIA. Comparing with eq.(3.6), (3.7) of [3] we see that in the present case there is no RR component in the boundary state, whereas the NSNS sector contribution has an extra factor of $\sqrt{2}$. This is consistent with the prescription of [6]. When we mod out by the transformation $\mathcal{I}_4$, θ is constrained to take value 0 or π, and we also need to add extra terms to the boundary state which will be responsible for projecting out the $\mathcal{I}_4$ non-invariant states from the open string sector. These extra terms can also be described in the language of ref. [3]. The resulting boundary state describing the type IIA D-string on the orbifold is given by:

$$|\theta, \epsilon\rangle = \frac{1}{\sqrt{2}} |\theta, U\rangle_{NSNS} + \frac{1}{2}\epsilon(|T_1\rangle_{RR} + e^{i\theta}|T_2\rangle_{RR})\,, \tag{3.3}$$

where ϵ takes values ± 1 and $|T_1\rangle_{RR}$, $|T_2\rangle_{RR}$ are the RR components of the twisted sector boundary states located at $x^9 = 0$ and $x^9 = \pi$ respectively, as defined in [3]. Note that although in [3] we considered the case of type IIB string theory instead of type IIA, and considered orbifolding by $(-1)^{F_L}\mathcal{I}_4$ instead of just $\mathcal{I}_4$, these two differences compensate each other so that the states $|T_i\rangle_{RR}$ are valid states (*i.e.* satisfy the GSO projection) in the type IIA orbifold that we are analysing here.[5] Following the analysis of [3] one can easily verify that the effect of adding the twisted sector contribution to the boundary state is to add an extra projection operator $(1 + \mathcal{I}_4 \cdot (-1)^F)/2$ to the open string partition function with $\mathcal{I}_4$ acting as $(+1)$ and $(-1)^F$ acting as (-1) on the vacuum of the NS sector. Since for Chan Paton factor I the allowed states are $(-1)^F$ even, they must also be $\mathcal{I}_4$ even. On the other hand in the sector with Chan Paton factor σ_2 the allowed Fock space states are $(-1)^F$ odd and hence they must also be $\mathcal{I}_4$ odd. This is equivalent to assigning the vacuum $\mathcal{I}_4$ charge (-1) and keeping $\mathcal{I}_4$ even states in the spectrum. Thus the spectrum of open strings associated with the boundary state (3.3) agrees with the spectrum on the non-BPS string described earlier.

From (3.3) we see that the type IIA D-string on this orbifold acts as sources of the RR gauge fields in the twisted sector. Since these gauge fields couple to membranes wrapped on the collapsed 2-cycles, we see that the D-string on the orbifold can be regarded as membrane wrapped simultaneously on these two 2-cycles. Since both ϵ and $e^{i\theta}$ can take values ± 1, we can get four different configurations, representing four possible ways of wrapping the membrane around the two 2-cycles. Note however that in each case, the configuration is neutral under the untwisted sector RR gauge field A_μ which couples to the D0-brane charge of type IIA string theory.

Clearly the above procedure could be repeated even if we compactify the directions $x^6, \ldots x^8$ so that the orbifold describes a K3 manifold. Thus the procedure described here gives us a solvable boundary conformal field theory describing membranes wrapped on non-supersymmetric cycles of K3. However in this case the Fock space ground state of the open strings with ends on the D-string and carrying one unit of winding number along x^6, x^7 or x^8, is not projected out, as $\mathcal{I}_4$ maps a winding number 1 state to a winding number -1 state. If we consider the state with one unit of winding number along x^8 for definiteness, then the mass m of this state is given by:

$$m^2 = R_8^2 - \frac{1}{2}\,. \tag{3.4}$$

[5]This can be seen by noting that the GSO projection operators in the closed string sector of the present theory, acting on the twisted sector RR ground state, takes the same form as given in eq.(2.41) of [3]. The T-dual version of this result has been discussed in [4].

Thus in order that there are no tachyonic excitations, we need to have $R_8 \geq \frac{1}{\sqrt{2}}$. Similarly by analysing the winding modes along x^6 and x^7 we get similar constraints on R_6 and R_7. Thus the membrane wrapped on the non-BPS cycle is stable in the region:

$$\begin{aligned} R_6 &\geq \frac{1}{\sqrt{2}}, \qquad R_7 \geq \frac{1}{\sqrt{2}}, \\ R_8 &\geq \frac{1}{\sqrt{2}}, \qquad R_9 \leq \sqrt{2}. \end{aligned} \tag{3.5}$$

As we have already seen, beyond the boundary $R_9 = \sqrt{2}$, the state can decay into a pair of membranes wrapped on supersymmetric cycles. In order to see what happens at $R_8 = (1/\sqrt{2})$, we note that this configuration is related to the one at $R_9 = \sqrt{2}$ by a T-duality transformation $R_8 \to (1/R_9)$, $R_9 \to (1/R_8)$. Since this T-duality transformation maps a supersymmetric cycle to a supersymmetric cycle, we see that even beyond the boundary at $R_8 = (1/\sqrt{2})$ the non-BPS state becomes unstable against decay into a pair of D-branes wrapped on supersymmetric cycles of K3. The same result holds at the boundaries at $R_6 = (1/\sqrt{2})$ and $R_7 = (1/\sqrt{2})$. Note however that the pair of supersymmetric cycles involved are different beyond different boundaries.

Instead of starting with a non-BPS string of type IIA string theory, we could have started with a non-BPS $(2p+1)$-brane of type IIA string theory with $(p \leq 2)$, with one of the tangential directions along x^9, and the other directions along $x^1, \ldots x^{2p}$.[6] After modding out by $\mathcal{I}_4$ this describes a stable $2p$-brane along non-compact directions in the range of parameters given in (3.5). Following the same analysis given in this section, one can easily see that this describes a BPS $(2p+2)$-brane of type IIA string theory, wrapped on a non-supersymmetric 2-cycle of K3. Similarly, if we start with a non-BPS D-$2p$-brane of type IIB string theory stretched along $x^9, x^1, \ldots x^{2p-1}$ $(p \leq 3)$ and mod out the theory by $\mathcal{I}_4$, we shall get a D-$(2p+1)$-brane of type IIB, wrapped on a non-supersymmetric 2-cycle of K3. The case where more than one tangential directions of the brane are along the compact directions $x^6, \ldots x^9$ will be discussed in section 5.

4. World-volume theory on coincident D-branes

In this section we shall discuss the world-volume theory of the D-branes constructed in section 3. For definiteness we shall carry out the analysis for a non-BPS 4-brane of type IIB stretched along $x^1, \ldots x^3, x^9$. After compactification of $x^6, \ldots x^9$ directions and modding out by $\mathcal{I}_4$, this will give rise to a type IIB D5-brane wrapped on

[6] A non-BPS $(2p+1)$-brane of type IIA string theory is defined in the same way as a non-BPS D-string, except that we impose Neumann boundary condition along $(2p+1)$ spatial directions instead of only one spatial direction.

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a non-supersymmetric 2-cycle, with a (3+1) dimensional world-volume field theory. The results for other cases can be easily found from this via dimensional reduction/oxidation.

We begin with the case of a single D-brane of this kind. The spectrum of massless fields before projection by $\mathcal{I}_4$ can be easily found. The spectrum of states from the Chan Paton sector I is that of an $\mathcal{N}=4$ supersymmetric U(1) gauge theory, *i.e.* it has besides the gauge fields, four Majorana fermions and six real scalar fields. The spectrum of states from the Chan Paton sector σ_2 does not contain any massless bosonic states; however the Ramond sector gives another four massless Majorana fermions. Thus we have a U(1) gauge field, eight massless Majorana fermions and six scalars. Five of these six scalars correspond to the freedom of moving the brane along its transverse directions $x^4, \ldots x^8$, while the sixth scalar denotes the component of the gauge field on the 4-brane along x^9. Upon modding out the theory by $\mathcal{I}_4$, only two of the six scalars, corresponding to the freedom of moving the branes along x^4 and x^5 directions, survive. The gauge field as well as four of the eight Majorana fermions also survive the projection. Thus the final spectrum contains a gauge field, four Majorana fermions and two scalar fields.

We can now study the effect of bringing together N such branes on top of each other. If the branes are all identical (*i.e.* the parameters ϵ and θ appearing in (3.3) are identical for all branes) then the spectrum of open strings with two ends on two different branes is identical to that of open strings with both ends on the same brane. As a result the spectrum contains N^2 copies of all the fields that appear on the world volume of a single brane. They describe a non-supersymmetric U(N) gauge theory with four Majorana fermions and two scalars in the adjoint representation of the gauge group.

5. Some generalizations

5.1 Branes wrapped on other non-supersymmetric cycles of K3

In the last section we constructed non-BPS states in IIA on K3 by starting from a non-BPS string wrapped on a circle of the torus, and then modding out the theory by $\mathcal{I}_4$. But we could also have started with a non-BPS three brane of IIA wrapped on three of the circles of T^4 and modded out the resulting configuration by $\mathcal{I}_4$. For definiteness, let us assume that the three brane is wrapped along the 6-7-8 cycle. Since this configuration is related to the configuration discussed in section 3 by a T-duality transformation $R_i \to (1/R_i)$ for $6 \le i \le 9$, we do not need to carry out the analysis all over again. First of all, from (3.5) we see that this configuration is stable in the region:

$$R_6 \le \sqrt{2}\,, \qquad R_7 \le \sqrt{2}\,, \qquad R_8 \le \sqrt{2}\,, \qquad R_9 \ge \frac{1}{\sqrt{2}}\,. \tag{5.1}$$

In the analysis of section 3 we have seen that beyond the region of stability, the non-BPS state discussed there decays into a pair of BPS states obtained by wrapping BPS D-branes on supersymmetric 2-cycles of K3. Since the T-duality transformation discussed here maps a supersymmetric 2-cycle into a supersymmetric 2-cycle, we can conclude that even in the present case, the non-BPS state decays into a pair of BPS D2-branes wrapped on supersymmetric 2-cycles beyond the region of stability. The new supersymmetric cycles are related to the ones in section 3 by the T-duality transformation $R_i \to (1/R_i)$ for $6 \le i \le 9$. Thus the non-BPS configuration obtained by wrapping a 3-brane on the 6-7-8 direction can be interpreted as a D2-brane wrapped on the non-supersymmetric cycle which is homologically identical to the sum of the two supersymmetric 2-cycles into which it decays beyond the region of stability.

As in section 3, instead of starting with a non-BPS 3-brane, we could have started with a non-BPS $(2p+3)$ brane stretched along $x^6, x^7, x^8, x^1, \ldots x^{2p}$ $(p \le 2)$. After modding out by $\mathcal{I}_4$ this configuration can be interpreted as a BPS $(2p+2)$ brane of IIA, wrapped on the non-supersymmetric cycle discussed above.

5.2 Branes wrapped on non-supersymmetric 2- and 3-cycles of a Calabi-Yau manifold

In this section we shall discuss construction of branes wrapped on non-supersymmetric 2- and 3-cycles of a Calabi-Yau manifold. The specific Calabi-Yau manifold that we shall be considering is the one discussed in [9]. We compactify type IIA string theory on T^6 labelled by coordinates $x^4, \ldots x^9$, and mod out the theory by a $Z_2 \times Z_2$ symmetry, generated by $\mathcal{I}_4$ discussed in section 3, together with

$$\mathcal{I}_4' : \left(x^4, \ldots x^9\right) \to \left(-x^4, -x^5, -x^6 + \pi R_6, -x^7, x^8 + \pi R_8, x^9\right). \tag{5.2}$$

Here R_n denotes the radius of the circle spanned by x^n. This model can be regarded as the result of modding out by $\mathcal{I}_4'$ the product of the K3 orbifold discussed in section 3 and the two dimensional torus T^2 spanned by x^4, x^5. Since $\mathcal{I}_4'$ includes a shift by πR_8 along x^8, there are no fixed points on T^6 under $\mathcal{I}_4'$. Similarly one can check that there are no fixed points of the transformation $\mathcal{I}_4\mathcal{I}_4'$ either. Thus the non-trivial cycles of the Calabi-Yau orbifold are obtained by taking $\mathcal{I}_4'$ invariant cycles on the product of T^2 and the K3 orbifold discussed in the last section. In particular if C denotes a two cycle of K3, C' denotes its image under the part of $\mathcal{I}_4'$ that acts on $x^6, \ldots x^9$, S^1 denotes a 1-cycle on T^2 labelled by x^4, x^5, and $S^{1\prime}$ denotes its image under the part of $\mathcal{I}_4'$ acting on T^2, then $C + C'$ gives a 2-cycle of the Calabi-Yau manifold, and $(C \times S^1 + C' \times S^{1\prime})$ gives a 3-cycle of the Calabi-Yau manifold.

In order to construct a non-BPS D-brane configuration in this theory, we must start with a non-BPS D-brane configuration on the original torus which is invariant under $\mathcal{I}_4$ and $\mathcal{I}_4'$. The non-BPS D-string lying along x^9 at

$$\begin{aligned} x^i &= a^i \qquad\qquad \text{for } 1 \le i \le 5\,, \\ x^6 &= x^7 = x^8 = 0\,, \end{aligned} \tag{5.3}$$

is invariant under $\mathcal{I}_4$, but transforms under $\mathcal{I}_4'$ to another non-BPS D-string along x^9, located at,

$$\begin{aligned} x^i &= a^i \qquad\qquad \text{for } 1 \le i \le 3\,, \\ x^i &= -a^i \qquad\qquad \text{for } i = 4,\, 5\,. \\ x^6 &= \pi R_6, \qquad x^7 = 0\,, \qquad x^8 = \pi R_8\,, \end{aligned} \tag{5.4}$$

Here a^i are arbitrary constants. Thus if we start with a pair of non-supersymmetric D-strings in type IIA on T^6 located at (5.3), (5.4), and mod out the theory by the $Z_2 \times Z_2$ symmetry, we get a non-BPS state of type IIA string theory on a Calabi-Yau orbifold. The boundary states corresponding to the two D-strings must be chosen in such a way that they are mapped to each other under the action of $\mathcal{I}_4'$. This relates the ϵ and θ parameters of the two D-strings. Given the boundary states, the spectrum of open strings with ends on the D-string or its image under $\mathcal{I}_4'$ can be computed in a straightforward manner.

The physical interpretation of this stable non-BPS state is straightforward. From our analysis of section 3 we know that after modding out by $\mathcal{I}_4$, the D-string lying at (5.3) denotes a membrane wrapped on a non-supersynmmetric cycle C. Since (5.4) is the image of (5.3) under $\mathcal{I}_4'$, the D-string lying at (5.4) denotes a membrane wrapped on C', where C' is the image of C under $\mathcal{I}_4'$. Thus the combined system of D-strings denotes a membrane wrapped on $C + C'$, which, according to our previous analysis, is a 2-cycle of the Calabi-Yau manifold. Thus the D-string configuration considered here denotes a membrane wrapped on a non-supersymmetric 2-cycle of the Calabi-Yau manifold.

Instead of starting with a D-string, we could have started with a pair of D3-branes along x^9, x^1, x^2, located at

$$\begin{aligned} x^i &= a^i \qquad\qquad \text{for } 3 \le i \le 5\,, \\ x^6 &= x^7 = x^8 = 0\,, \end{aligned} \tag{5.5}$$

and

$$\begin{aligned} x^3 &= a^3\,, \\ x^i &= -a^i \qquad\qquad \text{for } i = 4, 5\,. \\ x^6 &= \pi R_6, \qquad x^7 = 0, \qquad x^8 = \pi R_8\,, \end{aligned} \tag{5.6}$$

This describes a 2-brane in the non-compact directions and has the interpretation of a type IIA 4-brane wrapped along a non-supersymmetric 2-cycle of the Calabi-Yau manifold.

Instead of taking the pair of three branes to be lying parallel to the 1-2-9 plane, we could have taken them to be parallel to the 1-5-9 plane, and located at

$$\begin{aligned} &x^i = a^i \qquad \text{for } 2 \leq i \leq 4\,, \\ &x^6 = x^7 = x^8 = 0\,, \end{aligned} \tag{5.7}$$

and

$$\begin{aligned} &x^i = a^i \qquad \text{for } i = 2,\, 3\,, \\ &x^4 = -a^4\,, \\ &x^6 = \pi R_6, \qquad x^7 = 0, \qquad x^8 = \pi R_8\,. \end{aligned} \tag{5.8}$$

In this case, before the $\mathcal{I}'_4$ projection, the two D-branes have the interpretation of a D4-brane of type IIA wrapped on $C \times S^1$ and $C' \times S^{1\prime}$ respectively, with S^1, $S^{1\prime}$ denoting a one cycle on T^2 along the x^5 direction and its image under $\mathcal{I}'_4$ respectively. Thus the combined system denotes a type IIA 4-brane wrapped on $(C\times S^1 + C'\times S^{1\prime})$. From our previous discussion we see that after modding out the theory by $\mathcal{I}'_4$ this denotes a type IIA 4-brane wrapped on a non-supersymmetric 3-cycle of the Calabi-Yau manifold.

6. Descent relations among D-branes

The spectrum of D-branes in type IIA string theory contains BPS $2p$ branes and non-BPS $(2p+1)$ branes. Type IIB string theory on the other hand contains BPS $(2p+1)$-branes and non-BPS $2p$-branes. Using the results of section 2, and of [5], we get the following set of relations between different D-branes in type II string theory. For definiteness we shall focus on type IIA string theory, and start with a coincident $2p$-brane - anti-$2p$-brane pair. There is a complex tachyon field living on the world-volume of this system. If the tachyon condenses to the minimum value T_0 of the tachyon potential everywhere, then this system is indistinguishible from vacuum. If instead we consider a tachyonic 'kink' solution that is independent of time, as well as $(2p-1)$ of the spatial coordinates on the world-volume, and has the following behaviour as a function of the remaining world-volume coordinate x:

$$\begin{aligned} &T(x) \to T_0 \qquad \text{as} \quad x \to \infty\,, \\ &T(x) \to -T_0 \quad\ \text{as} \quad x \to -\infty\,, \end{aligned} \tag{6.1}$$

then it describes a non-BPS D-$(2p-1)$ brane of type IIA string theory [5]. This non-BPS brane in turn has a real tachyon field $\widetilde{T}$ living on it, At the minimum $\widetilde{T}_0$ of the tachyon potential the system is again indistinguishible from the vacuum. However if we consider a time independent kink solution associated with this new tachyon which is independent of $(2p-2)$ of the spatial directions on the world-volume and behaves as

$$\begin{aligned} \widetilde{T}(y) &\to \widetilde{T}_0 \qquad \text{as} \quad y \to \infty\,, \\ \widetilde{T}(y) &\to -\widetilde{T}_0 \quad \text{as} \quad y \to -\infty\,, \end{aligned} \tag{6.2}$$

y being the remaining world-volume coordinate, then it can be identified as the BPS $(2p-2)$-brane of the type IIA string theory.

A similar relationship holds between the BPS and non-BPS branes of type IIB string theory. Using these relations, all D-branes of type IIB string theory can be regarded as solitons on 9-brane – anti-9-brane system. This fact has been used recently to show that the D-brane charge takes value on the K-theory of space-time [7]. Similarly all D-branes of type IIA string theory can be regarded as solitons on an appropriate 8-brane anti-8-brane system.

In this section we shall discuss another kind of relationship between BPS and non-BPS D-branes. Our starting point will again be a brane-antibrane pair. Again for definiteness we shall consider a $2p$-brane - anti$-2p$-brane pair of type IIA string theory. But this time, instead of considering a tachyonic soliton on this pair, let us consider the effect of modding out the theory by $(-1)^{F_L}$. Since $(-1)^{F_L}$ changes the sign of all the RR sector fields, and since BPS D-branes carry RR charge, a BPS D-brane gets transformed to its antibrane under $(-1)^{F_L}$. As a result the brane-antibrane pair is invariant under $(-1)^{F_L}$ and it makes sense to mod out the configuration by $(-1)^{F_L}$.

In the bulk, modding out type IIA string theory by $(-1)^{F_L}$ gives type IIB string theory. The question we shall be interested in is: what happens to the brane-antibrane system under this modding? For this we need to compute the spectrum of open string states on the brane-antibrane system after the $(-1)^{F_L}$ projection. Before the projection, the open string states are labelled by 2×2 Chan Paton factors. If we take the basis of Chan Paton factors to be the identity matrix I and the three matrices σ_1, $i\sigma_2$ and σ_3, then, since I and σ_3 correspond to open strings with both ends lying on the D-brane (anti-D-brane), they contain Fock space states which are even under $(-1)^F$. On the other hand, σ_1 and σ_2, representing open string states with one end on the D-brane and the other end on the anti-D-brane, contain Fock space states which are odd under $(-1)^F$. Since in the Neveu-Schwarz-Ramond formalism $(-1)^{F_L}$ does not act on any of the fields on the world-sheet of the fundamental string, in order to determine the spectrum of $(-1)^{F_L}$ invariant open string states we only need to determine its action on the Chan-Paton factor Λ. Since $(-1)^{F_L}$ maps the brane to the antibrane, its action on the Chan Paton factors takes the form:

$$\Lambda \to S \Lambda S^{-1}\,, \tag{6.3}$$

where S can be taken to be either σ_1 or σ_2. Both choices are equivalent; for definiteness let us choose S to be σ_2. Thus only those open string states for which Λ

commutes with σ_2 survive the projection. This gives:

$$\Lambda = I\,, \sigma_2\,. \tag{6.4}$$

Since the sector I contains $(-1)^F$ even Fock space states, and the sector σ_2 contains $(-1)^F$ odd Fock space states, we see that the spectrum of open strings after the $(-1)^{F_L}$ projection agrees precisely with the spectrum of a non-BPS D-$2p$-brane of type IIB string theory. Thus we conclude that the D-$2p$ anti-D-$2p$-brane pair of type IIA string theory, after being modded out by $(-1)^{F_L}$, gives a non-BPS D-$2p$-brane of type IIB string theory. A similar analysis shows that the D-$(2p+1)$ anti-D-$(2p+1)$-brane pair of type IIB string theory, after being modded out by $(-1)^{F_L}$, gives a non-BPS D-$(2p+1)$-brane of type IIA string theory.

Let us now consider the effect of modding out the non-BPS D-$2p$-brane of type IIB string theory by $(-1)^{F_L}$. This is a possible operation, since this $2p$-brane does not carry any RR charge, and hence is invariant under $(-1)^{F_L}$. In the bulk, the theory goes back to type IIA string theory. In order to see what happens to the D-brane, we need to find the spectrum of open strings after the $(-1)^{F_L}$ projection. As discussed earlier, $(-1)^{F_L}$ does not act on the fields on the world-sheet of the fundamental string, and hence we only need to find its action on the Chan Paton factors. It turns out that $(-1)^{F_L}$ leaves states in the Chan Paton sector I invariant, and reverses the sign of the states in the Chan Paton sector σ_2. This can be seen by noting that *(i)* the two point function of the anti-symmetric tensor field $B_{\mu\nu}$ in the NSNS sector of the closed string, and the gauge field $\mathcal{A}_\mu$ arising in the Chan Paton sector I of the open string, is non-vanishing, and *(ii)* the two point function of the $2p$-form field $A^{(2p)}$ arising in the RR sector of the closed string, and the tachyonic open string state arising in the Chan Paton sector σ_2, is non-vanishing. Since $B_{\mu\nu}$ is even under $(-1)^{F_L}$, so must be states in the Chan Paton sector I. On the other hand since $A^{(p)}$ is odd under $(-1)^{F_L}$, so must be the states in the Chan Paton sector σ_2. Thus after the $(-1)^{F_L}$ projection we are only left with states in the Chan Paton sector I. This sector contains Fock space states which are even under $(-1)^F$. This spectrum agrees with the spectrum of open strings with both ends on a D-$2p$-brane of type IIA string theory. Thus we conclude that the result of modding out the type IIB D-$2p$-brane by $(-1)^{F_L}$ is a BPS D-$2p$-brane of type IIA string theory. A similar analysis shows that the result of modding out the type IIA D-$(2p+1)$-brane by $(-1)^{F_L}$ is a BPS D-$(2p+1)$-brane of type IIB string theory.

These results have been summarized in fig. 2.

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